Pitch Intervals - Relative Pitch Evaluation Example

Richard Fridrich

Introduction

- This presentation expands on the topic of pitch intervals and relative pitch presented in SAE paper 2003-01-1503.
- The measurement method deals with human perception of the variation in speed of an electric motor as changes in pitch.
- The intent is illustrate the development of a Sound Quality specification for electric motor operation.

Introduction

- The following slides provide a short summary of the concept and formulae used to calculate relative pitch interval changes from frequency (Hz) or motor speed (rpm) values.
- Then an example is presented using the measured motor speed of a sunroof as its sunshade is closed.

Basic Concepts

- Pitch is well defined in music using the Equal Tempered Scale
- The frequencies and intervals in the scale are internationally standardized
- Borrowing pitch concepts and definitions from music for product sound quality purposes saves time and resources

Musical Concepts

- In music the octave interval is divided into a scale of twelve intervals of equal pitch called half-steps.
- The frequencies associated with the pitches in the scale follow a logarithmic pattern.
- When preparing to perform, musicians tune their instruments using a finer scale called cents that divides a half-step into 100 increments.
- In performance, musicians generally discuss pitch intervals in terms of “half-steps” or “whole-steps” not cents. (For SQ either can be used)

Pitch Interval Formulae

$$PitchInterval = M * \log_{10}\left(\frac{f\ final}{f\ initial}\right)$$

General Equation

$$PitchInterval = 19.9317 * \log_{10}\left(\frac{f\ final}{f\ initial}\right)$$

Units: Scale-steps

$$PitchInterval = 3986.337 * \log_{10}\left(\frac{f\ final}{f\ initial}\right)$$

Units: Cents

Cents = 200 * Scale-steps

Unit Conversion

Advantages (Scale-steps or Cents)

- Equal Tempered Scale = Standardized
- “Half-step” Terminology = Standardized
- “Cents” Concept = Standardized
- “Scale-steps” units links to Musical Vocab.
- Simple Conversion: Scale-steps-to-Cents
- Calculated Magnitude = Pitch Interval
- Calculated Sign = Pitch Direction

Music Example Calculations

Range of Piano (88 keys => 87 half-steps => 43.5 whole steps):

$f_{initial}$ = A_0 = 27.5 Hz to f_{final} = C_8 = 4186.0 Hz

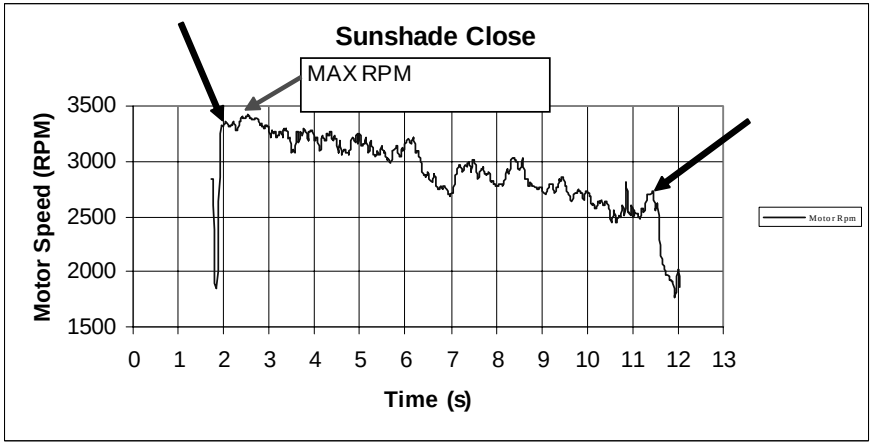
$$PitchInterval = 19.9317 * \log_{10}\left(\frac{f\ final}{f\ initial}\right)$$ 43.500 scale-steps

$$PitchInterval = 3986.337 * \log_{10}\left(\frac{f\ final}{f\ initial}\right)$$ 8700.0 cents

Application Example

- Setting Specification Values requires benchmarking similar systems and subjectively rating them acceptable and unacceptable.
- Three Recommended benchmarking measurement quantities are:
  - Absolute Maximum RPM
  - Relative Pitch Slope
  - Variation Allowance

Application Example



Example is the sunshade closing operation of a power sunroof.

Export the motor speed time history to an EXCEL spreadsheet.

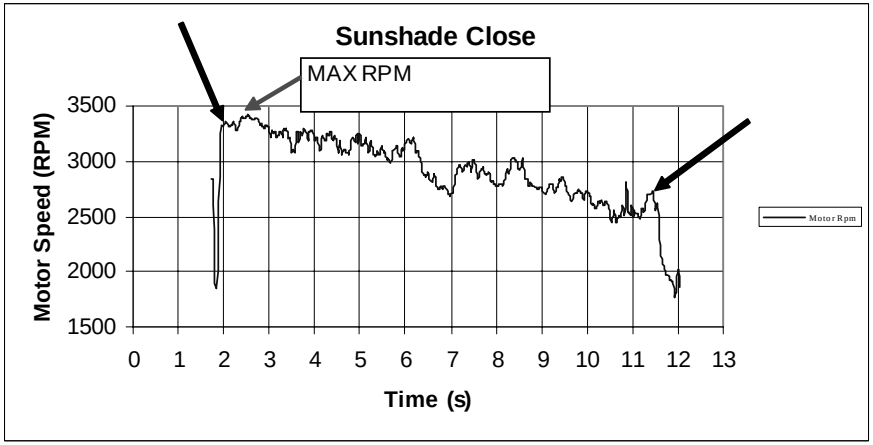
Select Time Interval: (defined by black arrows)

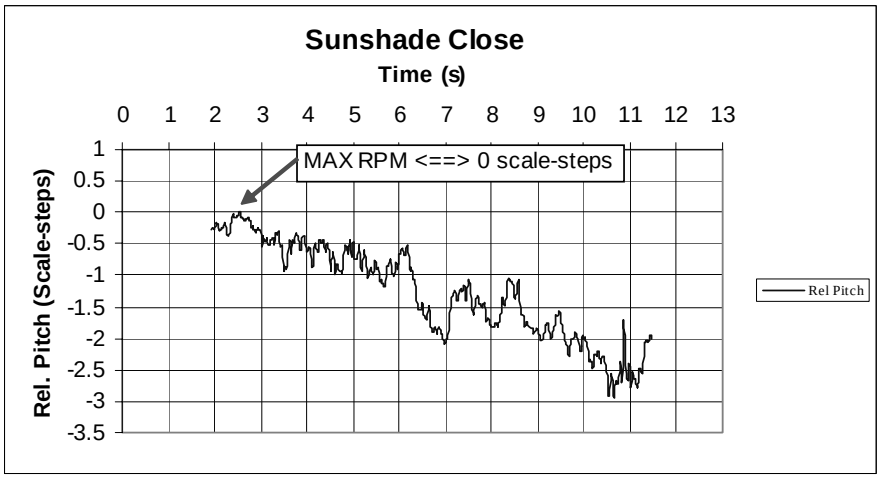
Find MAX RPM:

In this case,

MAX RPM = 3421 rpm

Application Example





Calculate the Pitch intervals for each data point relative to the MAX RPM

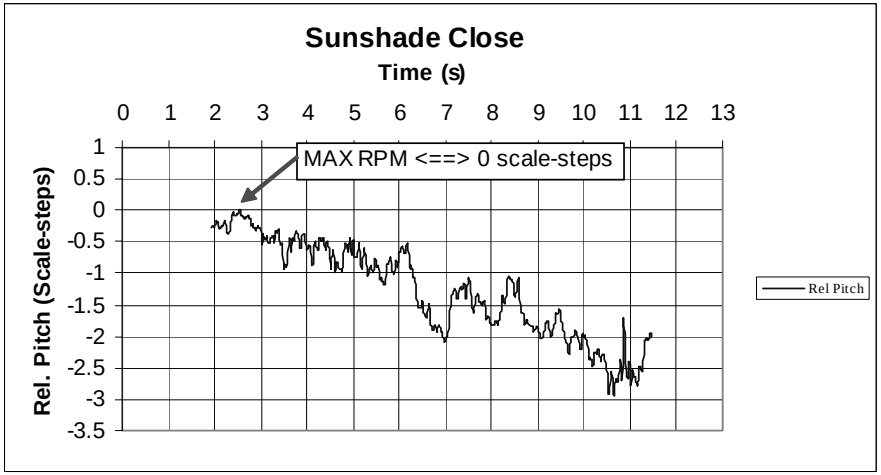
Set: $f_{initial}$ = MAX RPM

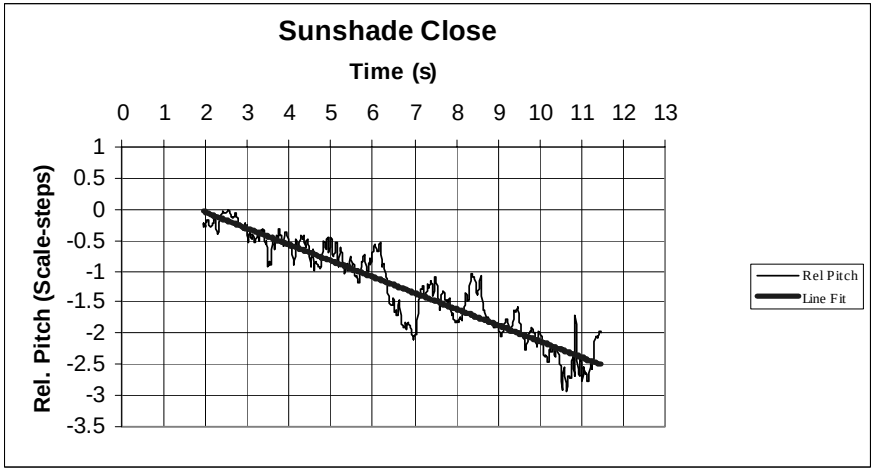
Set: f_{final} = RPM(t)

$$PitchInterval = 19.9317 * \log_{10}\left(\frac{f\,final}{f\,initial}\right)$$

Plot results:

Application Example





Determine Slope and Intercept using Linear Curve Fit

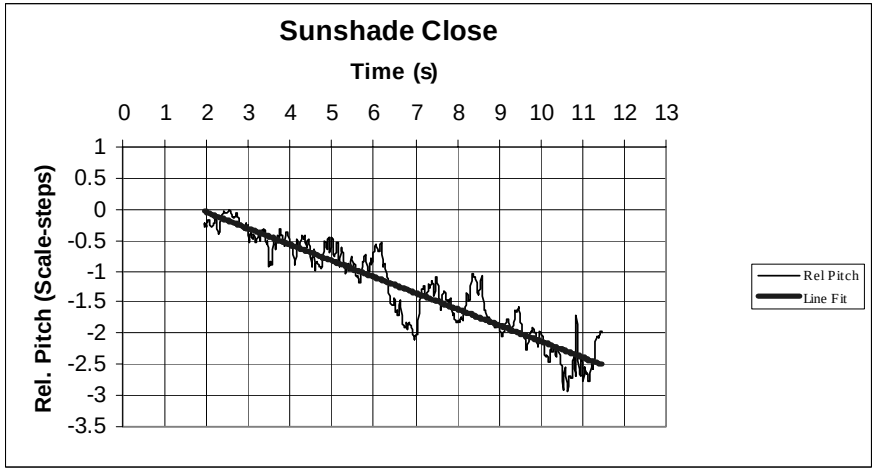
For this case,

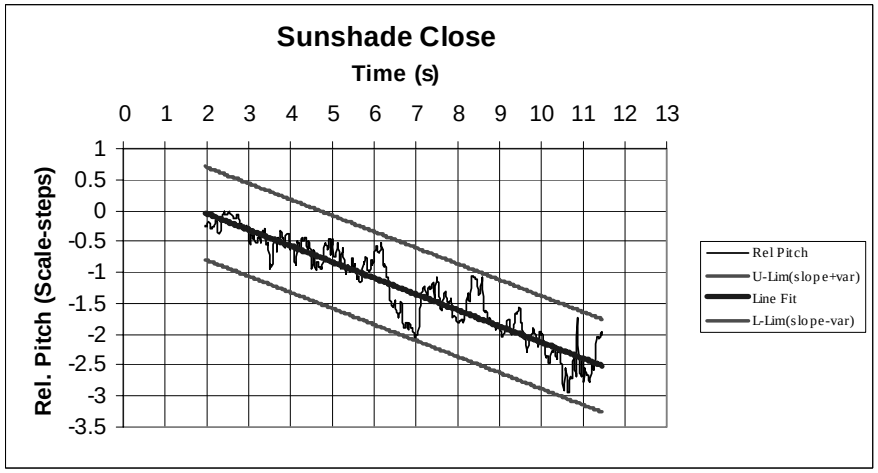
Slope = -0.259 scale-steps/s

Note that the Intercept has no significance for pitch perception. It is needed only for plotting the data.

Plot Curve Fit Line:

Application Example





Next, assess the pitch variation about the Curve Fit by Upper and Lower Limit Lines that completely contain the all the data.

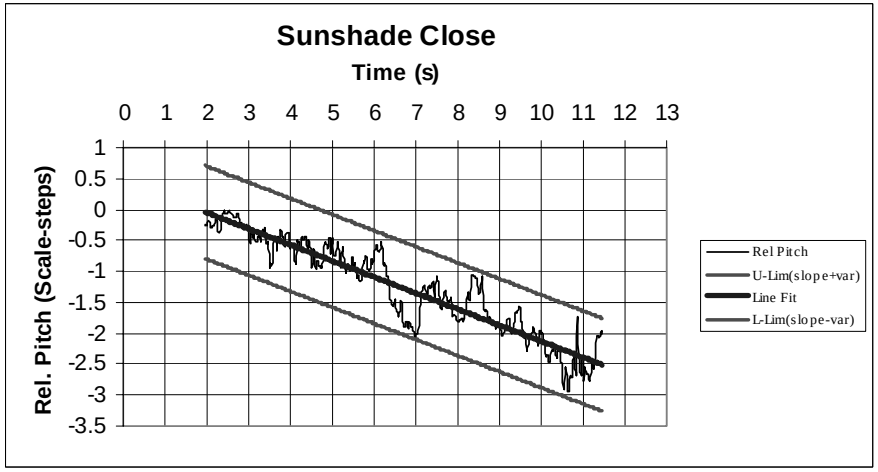
i.e. the Curve Fit Line +/- Pitch Variation Allowance

In this case,

Pitch Variation Allowance = 0.75 scale-steps

Plot Upper and Lower Limit Lines:

Application Example



RESULTS SUMMARY

MAX RPM = 3421 rpm

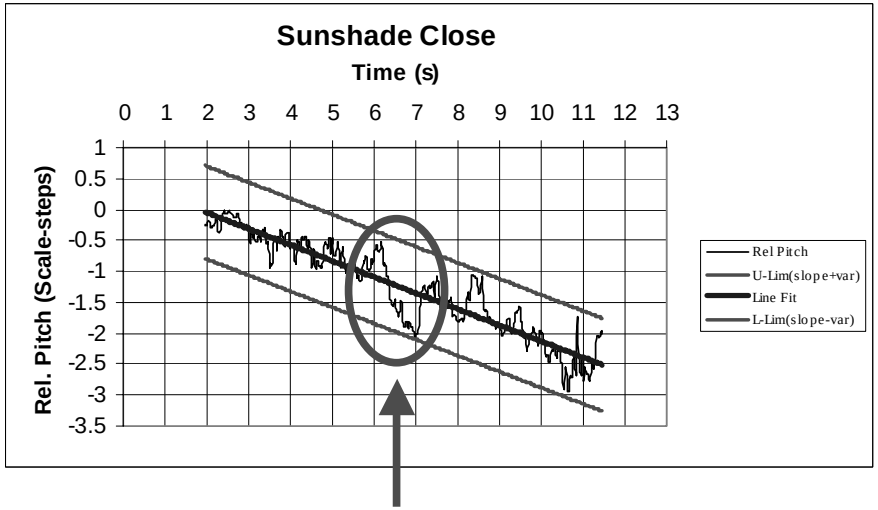
Slope = -0.259 scale-steps/s

Pitch Variation Allowance = 0.75 scale-steps

Including the MAX RPM measure in a specification is analogous to defining the first note of a melody in music. Once that note is defined as an absolute pitch, all the other notes of a melody are also defined in absolute terms.

Because both the Slope and Pitch Variation Allowance are relative measures, specifications using these measures can be applied to different electric motor products in a manner analogous to the transposing of a melody in music.

Discussion



RESULTS SUMMARY

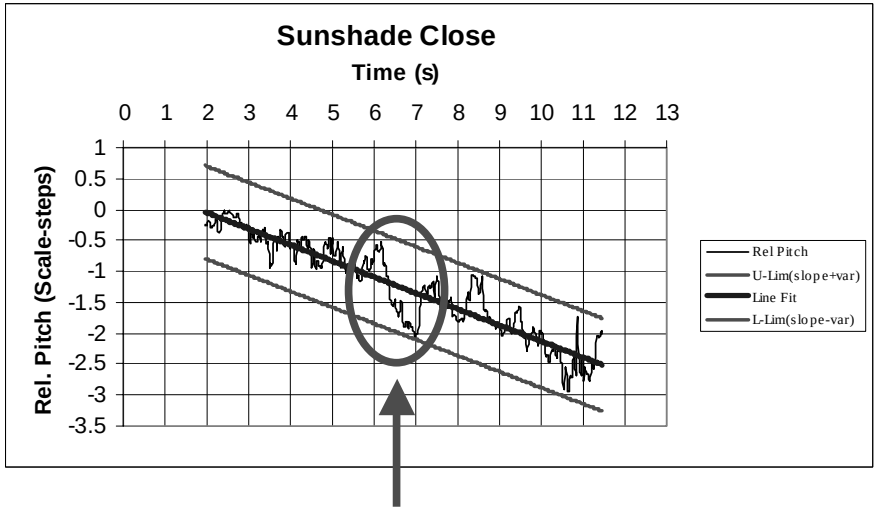
MAX RPM = 3421 rpm

Slope = -0.259 scale-steps/s

Pitch Variation Allowance = 0.75 scale-steps

One particular aspect illustrated by this example was found to be objectionable -- the rather large variation in pitch that occurs near the middle of the travel time. Benchmarking other examples can help answer the question of how much pitch variation is acceptable. With some luck the benchmarking work may already have been done and just needs to be reanalyzed.

Discussion



RESULTS SUMMARY

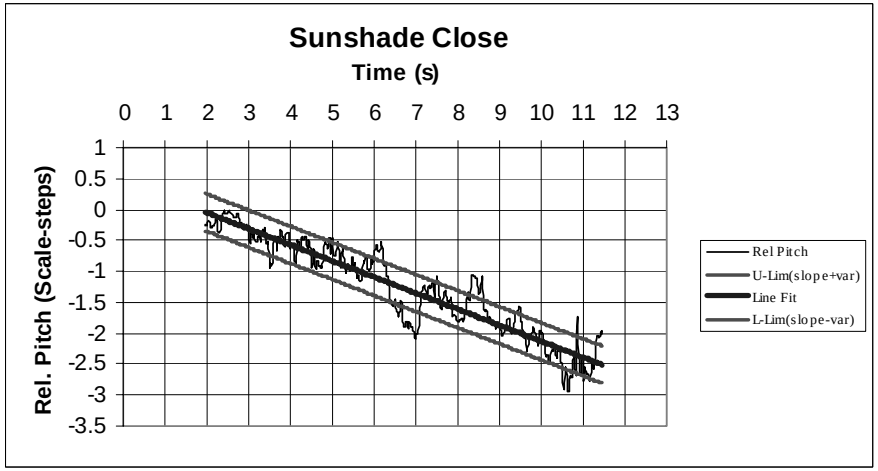
MAX RPM = 3421 rpm

Slope = -0.259 scale-steps/s

Pitch Variation Allowance = 0.75 scale-steps

A review of some early work by a former colleague (who had used absolute RPM measures in his study rather than the relative pitch techniques) seemed to indicate that an electric motor sound with a slope of +/–0.260 scale-steps/s was generally rated acceptable provided the pitch variation was not too great. His work concerning acceptable pitch variation translated to a Pitch Variation Allowance of 0.3 scale-steps. The absolute MAX RPM range of his study was from 3000 rpm to 5000 rpm. These results are applied on the next slide.

Discussion



RESULTS SUMMARY

MAX RPM = 3421 rpm

Slope = -0.259 scale-steps/s

Pitch Variation Allowance = 0.75 scale-steps

This graph evaluates the sunshade example using the Pitch Variation Allowance of 0.3 scale-steps as a specification limit. It illustrates where during the sunshade travel time the variation becomes excessive. The Pitch Variation is the only aspect that was determined to be objectionable since the MAX RPM and Slope values of the sunshade example are within the range previously found acceptable by the Benchmark study. Using the Relative Pitch method and Benchmarking results (derived from the work of a former colleague) an example specification is presented on the next slide.

Example Specification

Absolute Maximum RPM Requirement

(The maximum RPM shall be greater than 3000 rpm but less than 5000 rpm)

Relative Pitch Slope Requirement

(During the sunshade close operation, the slope of the relative pitch shall be greater than -0.26 scale-steps/s but less than +0.26 scale-steps/s)

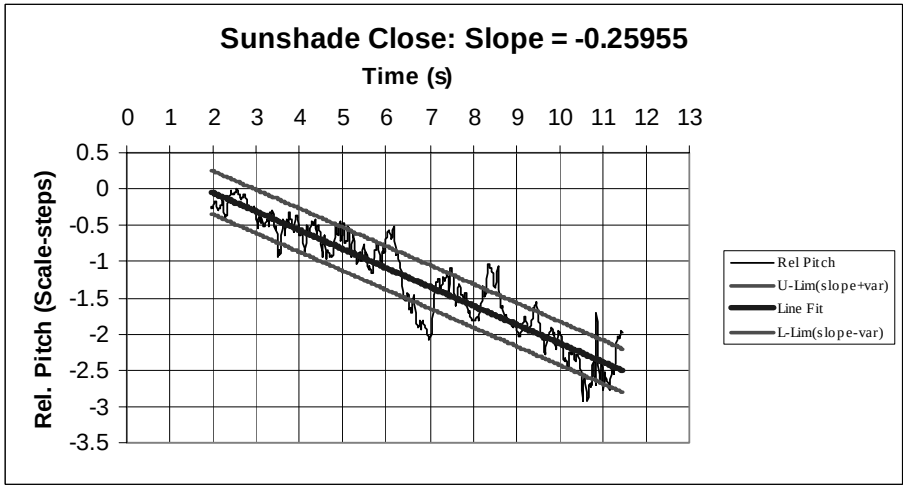
Variation Allowance Requirement

(During the sunshade close operation, the allowable relative pitch variation about the slope shall not exceed +/- 0.3 scale-steps)

Note: In music it is often more important to define a melody by changes in relative pitch rather than absolute pitch. This allows the music to be transposed to other keys. In Product Sound Quality, the relative pitch slope and variation allowance requirements are also likely to be more important than the absolute maximum RPM requirement because then requirements developed for one system are more likely to be applicable to other similar systems.

Application Example Summary

Evaluate for Compliance with Specification



✓Sunshade Meets Absolute Max RPM Requirement

✓Sunshade Meets Slope Requirement >-0.26

X - Verify Relative Pitch data does not exceed Upper and Lower Limits.

[Sunshade Exceeds Variation Allowance Requirement]

Application Example

REMINDER

Pitch Interval Method is NOT LIMITED to RPM

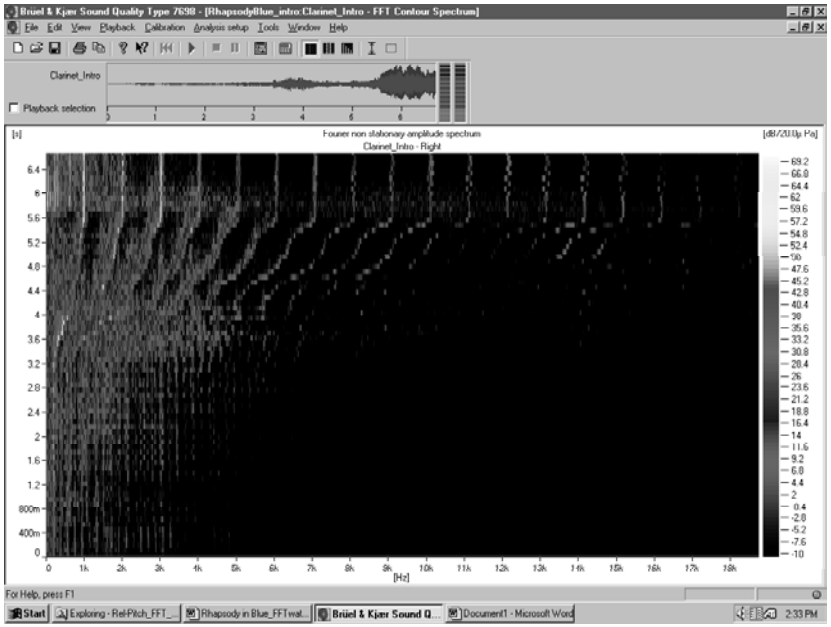
Input can also be

Frequency Component Time History

(Order Tracking)

Application Example

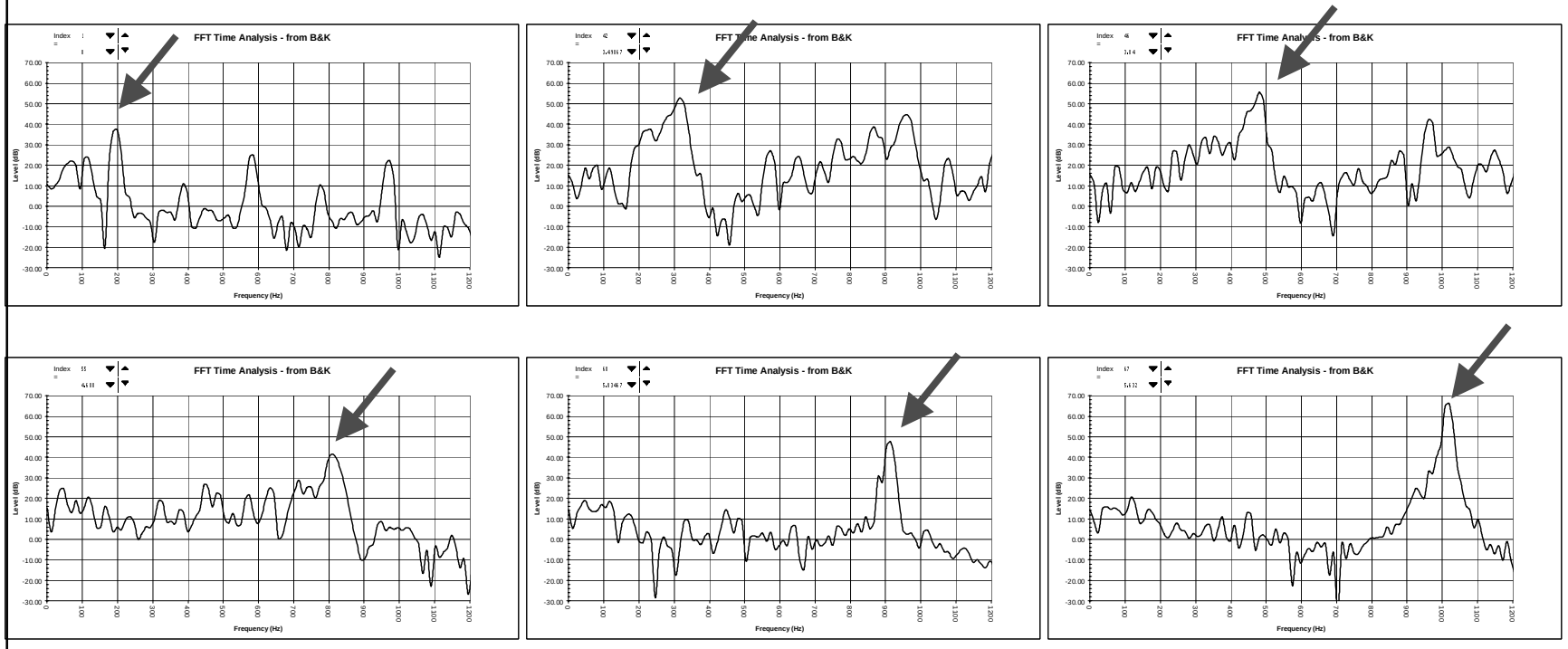
Tone Tracking using FFT Waterfall



Obtain information on Relative Pitch changes by extracting frequency-time history for any single harmonic or order.

Application Example

FFT Waterfall exported to MS-Excel
(Selected Plots to Illustrate Tracking Frequency Peaks →)



Extrapolating Requirements

- In thinking about how to write requirements to improve performance from merely acceptable to excellent, consider how to reduce variation:
  - For a constant running motor, the Relative Pitch Slope requirement should approach zero.
  - The Pitch Variation Allowance requirement likely should be smaller than a half-step total (+/- 0.25), but there is a practical limit to the pitch variation that is audible [i.e. just noticeable difference (jnd) at medium to high frequencies ~ 0.06 scale-steps (calculated from 0.7% jnd for frequency variation reported by Zwicker & Fastl)]. Determination of the jnd is performed under laboratory conditions, so an allowance of 0.06 scale-steps is probably too small for most real-world conditions. An allowance of 0.12 scale-steps is probably a reasonable allowance requirement to consider trying.

Final Summary

•Pitch intervals and relative pitch are proven concepts from music that relate frequency to human perception.

•Motor Speed or Frequency Component Time Histories can be used to evaluate for:
•Absolute Maximum RPM or Frequency
•Relative Pitch Slope
•Variation Allowance

•Relative pitch slope requirements and variation allowance requirements developed for one system are likely to be applicable to other similar systems

•Borrowing pitch concepts and definitions from music for product sound quality purposes saves time and resources

End of Presentation